Indian Journal of Sci. Research: 2026 SPECIAL EDITION ISSN: 0976-2876 (Print)

Proceeding of R.B.G.P. 21st Century Edu.,Eco.,Sci. & Tech. DOI: 10.32606 ISSN: 2250-0138 (Online)

MATHEMATICAL MODEL FOR KINETIC ALFVEN WAVES IN THE PRESENCE OF LOSS CONE DISTRIBUTION FUNCTION IN INHOMOGENEOUS MAGNETOPLASMA FOR MULTI-IONS PLASMA

MAHENDRA AHIRWAR[a], B.D. RAIKWAR[b] AND RADHA TAMRKAR[c1]

[a]Department of Physics, Maharaja Chhatrasal Bundelkhand University, Chhatarpur, M.P., India
[b]Department of Physics, PMCOE, Govt. P.G. College, Damoh, M.P., India
[c]Department of Physics, Govt Kamla Nehru Mahila Mahavidyalaya, Damoh, M.P., India

ABSTRACT

This work presents a mathematical model for the propagation of Kinetic Alfven waves in an inhomogeneous magnetized plasma containing multiple ion species under the influence of a loss-cone distribution function. In space plasma environments, variations in magnetic field strength and plasma density significantly affect the behavior of electromagnetic waves and their interaction with charged particles. In this study, these inhomogeneities are incorporated into the analysis to examine their influence on wave dispersion and resonance processes. The model also considers finite temperature effects and temperature anisotropy, which play an important role in determining the characteristics of Kinetic Alfven waves. The presence of a loss-cone distribution leads to strong anisotropy in velocity space, which can modify the stability of the wave by causing either amplification or attenuation depending on the pitch-angle distribution of particles. Particle motion is analyzed using the guiding-center approximation to investigate the exchange of energy between particles and electromagnetic field perturbations. The results contribute to a better understanding of wave–particle interaction mechanisms in space plasma systems such as the Earth's magnetosphere.

KEYWORDS: Kinetic Alfven Waves, Loss-Cone Distribution, Multi-Ion Plasma, Magnetized Plasma, Wave–Particle Interaction, Plasma Inhomogeneity, Finite Larmor Radius, Guiding-Center Approximation

INTRODUCTION

The dispersion characteristics, instability growth rate, and field-aligned current generation in the auroral acceleration region have previously been investigated within the theoretical framework of Kinetic Alfven waves (KAWs). Extending that investigation, the present study concentrates on a specific and physically significant problem, namely the instability of drift Kinetic Alfven waves produced by a loss-cone velocity distribution. The main objective is to explore how the existence and steepness of the loss-cone feature affect the dispersion relation and instability growth rate of electromagnetic drift Kinetic Alfven waves in an inhomogeneous magnetospheric plasma. In addition to the conventional single-ion approximation, the present analysis considers a more realistic multi-ion plasma consisting of hydrogen (H^+), helium (He^+), and oxygen (O^+) ions, which are commonly observed in the Earth's magnetospheric environment. Kinetic Alfven waves represent one of the fundamental electromagnetic wave modes in magnetized plasmas and play a crucial role in several plasma processes, including energy transport, particle acceleration, plasma heating, and the generation of field-aligned currents. These waves are also closely associated with the interpretation of ultra-low-frequency (ULF) emissions in the Earth's magnetosphere. Both high-latitude ULF phenomena and wave activities occurring from equatorial to mid-latitude magnetospheric regions have been successfully explained through the dynamics of Kinetic Alfven waves. Consequently, these waves are widely recognized as important agents in theoretical models describing magnetosphere–ionosphere coupling. Field-aligned currents form a fundamental component of magnetized plasma systems, particularly in processes that couple the magnetosphere with the ionosphere. Within the magnetohydrodynamic (MHD) approximation—applicable when the temporal and spatial scales of perturbations are much larger than the ion gyro period and ion gyro radius—any localized disturbance along a magnetic flux tube requires the development of field-aligned currents to transmit information along the entire magnetic field line. Such currents become especially significant in situations where different dynamical regions remain magnetically connected through the same field lines. Under these circumstances, plasma density inhomogeneities in the plasma sheet region can excite Kinetic Alfven waves, which then propagate along magnetic field lines toward the ionosphere. Earlier theoretical descriptions of Alfven waves in the magnetosphere were primarily based on simplified cold-ion assumptions within the MHD framework. However, such approximations are not always sufficient for realistic magnetospheric conditions. In the equatorial magnetosphere, ions often possess finite temperatures, and the corresponding Finite Larmor radius (FLR) effects of warm ions become important in determining the perpendicular wavelength of Kinetic Alfven waves. The inclusion of FLR corrections introduces the necessary dispersive behavior that limits the transverse scale length

[1]Corresponding author

of magnetohydrodynamic wave structures. As a result, a Kinetic treatment becomes essential for accurately describing short-wavelength Alfvenic phenomena in magnetospheric plasmas. The propagation and excitation of Kinetic Alfven waves in inhomogeneous plasmas have been investigated extensively in earlier theoretical studies. These investigations demonstrated that drift wave instabilities in finite-β plasmas are capable of exciting Alfvenic modes. Since drift waves typically possess perpendicular wavelengths comparable to the ion gyro radius, the excited electromagnetic mode essentially corresponds to a Kinetic Alfven wave. This concept was originally introduced by Hasegawa and Chen in the context of plasma heating processes. Subsequent investigations have examined the role of Kinetic Alfven waves in both homogeneous and inhomogeneous magnetic field configurations within magnetosphere–ionosphere coupling studies, largely using magnetohydrodynamic approaches. In particular, Leonovich and Mazur analyzed the propagation of Kinetic Alfven waves in two-dimensional inhomogeneous plasmas embedded in curved magnetic field geometries. Most of the existing theoretical analyses of Kinetic Alfven waves assume that plasma particles follow Maxwellian or bi-Maxwellian velocity distributions. Such assumptions neglect the presence of strong gradients that may arise in velocity space due to the formation of loss-cone features. In magnetospheric regions characterized by mirror-like magnetic configurations, such as the auroral zone where magnetic field lines are curved and converge toward the ionosphere, the particle distribution often deviates significantly from a Maxwellian form. In plasmas with low collisionality, strong anisotropies can naturally develop, giving rise to pronounced loss-cone distributions . The steepness of the loss-cone boundary plays an important role in wave–particle interactions and can considerably modify the growth and stability characteristics of plasma waves. Another important factor in magnetospheric plasma dynamics is the presence of multiple ion species. Observations indicate that, besides the dominant hydrogen ions (H^+), heavier ions such as helium (He^+) and oxygen (O^+) frequently exist in appreciable concentrations, particularly during disturbed geomagnetic conditions. The presence of these additional ion species introduces significant mass and charge effects that influence the dispersion properties, resonance conditions, and stability behavior of plasma waves. Therefore, incorporating a multi-ion plasma consisting of H^+, He^+, and O^+ ions provides a more realistic representation of the magnetospheric environment and allows a more comprehensive investigation of Kinetic Alfven wave dynamics. To examine these effects, an alternative theoretical framework known as particle aspect analysis is employed . This approach offers several advantages over conventional magnetohydrodynamic models, particularly in its ability to incorporate finite larmor radius effects, temperature anisotropy, and non-Maxwellian velocity distributions in inhomogeneous magnetoplasmas. In this formulation, the plasma is treated as a combination of resonant and non-resonant particle populations, and the instability growth is determined using the principle of energy conservation. The analysis considers kinetic Alfvén waves propagating obliquely with respect to a uniform background magnetic field directed along the z-axis. Two scalar potentials in the x–z plane are introduced to describe the perturbed electromagnetic fields responsible for modifying the trajectories of charged particles. The equilibrium plasma density gradient is assumed to be oriented along the y-direction. The presence of multiple ion species, namely H^+, He^+, and O^+, is incorporated into the formulation to account for realistic magnetospheric plasma composition and its influence on the propagation and instability characteristics of drift Kinetic Alfven waves. In the present analysis, Kinetic Alfven waves are assumed to propagate obliquely with respect to a uniform background magnetic field directed along the z-axis. Two scalar potentials defined in the x–z plane are used to describe the perturbed electromagnetic fields that influence the motion of charged particles. The equilibrium plasma density gradient is taken to be along the y-direction. The theoretical formulation explicitly incorporates a multi-ion plasma consisting of hydrogen, helium, and oxygen ions in order to provide a more realistic representation of the magnetospheric plasma composition and to examine how the presence of these ions affects the propagation and instability characteristics of drift Kinetic Alfven waves. The propagation of inertial Alfven waves in the auroral acceleration region is significantly influenced by the presence of multiple ion species such as H^+, He^+, and O^+. Numerical analysis indicates that the wave frequency varies approximately within the range of 0.5–18 s^{-1}, depending on the ion composition of the plasma. The calculated parallel and perpendicular group velocities are about 10^9 cm s^{-1} and 10^5 cm s^{-1}, respectively, demonstrating efficient energy transport along magnetic field lines. The characteristics of Kinetic Alfven waves in the plasma sheet boundary layer (PSBL) are strongly affected by the inclusion of heavy ions such as He^+ and O^+ in addition to H^+. The dispersion behavior was analyzed over a range of the perpendicular wave parameter $k_\perp \rho_i$, showing that the wave frequency decreases as the concentration of heavier ions increases. The instability properties were also found to depend on the loss-cone parameter, with noticeable differences between J = 1 and J = 2. The behavior of Kinetic Alfven waves in multi-ion plasma is considerably modified by the presence of a general loss-cone distribution function. Numerical results show that variations in the loss-cone index and heavy-ion density influence both the wave frequency and the growth characteristics of the instability. The different gyroradii of H^+, He^+, and O^+ ions introduce noticeable changes in the dispersion properties of the wave.

BASIC TRAJECTORIES

We have created the particle aspect analysis for electromagnetic Kinetic Alfven waves in the current model in lowβ ($\beta=2\mu_o P/B_o^2$) a general loss-cone distribution function in an inhomogeneous plasma. The wave in an anisotropic plasma is thought to travel in the x-z plane, which is perpendicular to the density gradient. We use the method described in ref. (Tiwari and Varma 1993, 1991, Varma and Tiwari 1992, Terashima 1967) for the mathematical analysis.When the resonant particles are undisturbed, it is believed that the Kinetic Alfven wave begins at t=0. The Kinetic Alfven wave that meets the criterion is of interest to us.

$$V_{T\parallel i} << \frac{\omega}{K_\parallel} << V_{T\parallel e}, \quad \omega << \Omega_i, \Omega_e; \quad K_\perp^2\rho_e^2 << K_\perp^2\rho_i^2 \qquad \dots (1)$$

where $V_{T\parallel i}$ and $V_{T\parallel e}$ are the mean velocities of ions and electrons along the magnetic field, $\Omega_{i,e}$ are gyration frequencies and $\rho_{i,e}$ the mean gyroradii of the respective species.We have extended particle aspect analysis for electromagnetic perturbations in this chapter. In order to express electromagnetic perturbations in low-β plasmas, we have chosen two potential representations (Hasegawa and Chen 1975, 1976, Hasegawa and Mima 1978, Kadomtsev 1965). The compressional Alfven mode is supposed to be decoupled by assuming, $(\nabla \times E)_\parallel = -\frac{\partial B_z}{\partial t} = 0$, that is taking into account only the effect of the field line bending. This enables one to use a scalar potential $\emptyset$ to express the perpendicular components of the wave electric field,

$$E_\perp = -\nabla_\perp \emptyset$$

Because the wave is electromagnetic, however, we have to use $E_\parallel$ which is not equal to $-\frac{\partial \emptyset}{\partial z}$. For $E_\parallel$ we hence use a different potential, Ψ;

$$E_\parallel = -\nabla_\parallel \Psi,$$

and $\emptyset \neq \Psi$. The potentials $\emptyset$ and Ψ must satisfy suitable field equations.We begin by considering the wave electric field E of the form

$$\bar{E} = \overline{E_\perp} + \bar{E}_\parallel$$

$$\emptyset = \emptyset_1 \cos(K_\perp x + K_\parallel z - \omega t)$$

$$\Psi = \Psi_1 \cos(K_\perp x + K_\parallel z - \omega t) \qquad \dots (2)$$

where $\emptyset_1$ and Ψ_1 are assumed to be a slowly varying function of time t, and ω is the wave frequency. The equation of motion of particle is

$$m\frac{d\bar{v}}{dt} = q\left(\bar{E} + \frac{1}{c}\bar{v} \times B_0\right) \dots (3)$$

where the collisions between particles are neglected. **v** can be expressed as a sum of the unperturbed velocity $\bar{V}$and the perturbed velocity $\bar{u}$ i.e. $\bar{v}=\bar{V} + \bar{u}$,$\bar{u}$ is determined by the following set of equations

$$\frac{du_+}{dt} + i\Omega u_+ = \frac{q}{m}\left[\emptyset_1 K_\perp - \frac{V_\parallel K_\perp K_\parallel}{\omega}(\emptyset_1 - \Psi_1)\right]\sin(K_\perp x + K_\parallel z - \omega t)$$

$$\frac{du_\parallel}{dt} = \frac{q}{m}\left[K_\parallel \Psi_1 + \frac{V_\perp K_\perp K_\parallel}{\omega}(\emptyset_1 - \Psi_1)\cos(\theta - \Omega t)\right]\sin(K_\perp x + K_\parallel z - \omega t) \dots (4)$$

where $u_+ = u_x + iu_y$, θ is the initial phase of velocity and $\Omega = q B_0/mcu_x$ and u_y are the perturbed velocities in the x and y directions respectively. Equation (4) is solved by replacing the co-ordinates of charged particles to that of free gyration (Tiwari and Varma 1993) which provides perturbed velocity $\bar{u}(t)$. The perturbed velocity $\bar{u}(t)$ can be further transformed to $\bar{u}(\bar{r}, t)$ by the use of trajectories of free gyration once again (Tiwari and Varma 1993). For the resonant particles it is necessary to take into account the initial condition $\bar{u}(\bar{r}, t=0) = 0$, inferred from the assumption stated above. Thus:

$$u_x(\bar{r}, t) = -\frac{q}{m}\left[\phi_1 K_\perp - \frac{V_\parallel K_\parallel K_\perp}{\omega}(\phi_1 - \Psi_1)\right]\sum_{-\infty}^{+\infty} J_n(\mu)\sum_{-\infty}^{+\infty} J_l(\mu)\left[\frac{\Lambda_n}{a_n^2}\cos\xi_{nl} - \frac{\delta}{2\Lambda_{n+1}}\cos(\xi_{nl} - \Lambda_{n-1}t) - \frac{\delta}{2\Lambda_{n-1}}\cos(\xi_{nl} - \Lambda_{n+1}t)\right]$$

$$u_y(\bar{r}, t) = -\frac{q}{m}\left[\phi_1 K_\perp - \frac{V_\parallel K_\parallel K_\perp}{\omega}(\phi_1 - \Psi_1)\right]\sum_{-\infty}^{+\infty} J_n(\mu)\sum_{-\infty}^{+\infty} J_l(\mu)\left[-\frac{\Omega}{a_n^2}\sin\xi_{nl} - \frac{\delta}{2\Lambda_{n+1}}\sin(\xi_{nl} - \Lambda_{n-1}t) + \frac{\delta}{2\Lambda_{n-1}}\sin(\xi_{nl} - \Lambda_{n+1}t)\right]$$

$$u_z(\bar{r},t) = -\frac{q}{m}\left[K_\parallel \Psi_1 + \frac{V_\perp K_\parallel K_\perp}{\omega}(\phi_1 - \Psi_1)\frac{n}{\mu}\right]\sum_{-\infty}^{+\infty} J_n(\mu) \sum_{-\infty}^{+\infty} J_l(\mu)\frac{1}{\Lambda_n}[\cos\xi_{nl} - \delta\cos(\xi_{nl} - \Lambda_n t)] \ldots.(5)$$

where $\delta = 0$ for the non-resonant particles and $\delta = 1$ for resonant one and

$$\Lambda_n = n\Omega + V_\parallel K_\parallel - \sum_i \omega_i \,, \mu = \frac{K_\perp V_\perp}{\Omega}$$

$$\xi_{nl} = (1-n)(\theta - \Omega t) + K_\perp x + K_\parallel z - \omega t \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$a_n^2 = \Lambda_n^2 - \sum_i {\Omega^2}_i$$

Also use was made of

$$\exp[-i\,\mu\sin(\theta - \Omega t)] = \sum_{-\infty}^{+\infty} J_n(\mu)\exp[-in(\theta - \Omega t)]$$

$$\cos\phi\,\exp(-i\;\mu\;\sin\phi) = \frac{n}{\mu}\sum_{-\infty}^{+\infty} J_n(\mu)\exp(in\phi)$$

The perturbed coordinates of the particles x, y, and z are obtained by integrating equation (5), which together with the free gyration trajectories show the particles actual journey. Given the initial estimates, the term n=0 makes up the majority of the contribution. The resonant criterion is given by $k_\parallel V_\parallel - \omega = 0$. Therefore, this resonance condition means that the electrons see the wave independent of t in the particles frame. The particles satisfying the above conditions are called resonant. $J_n(\mu)$ and $J_1(\mu)$ are Bessel's functions which arise from the different periodical variation of charged particle trajectories. The terms represented by Bessel's functions show the reduction of the field intensities due to finite gyroradius effect.

DISPERSION RELATION

To evaluate the dispersion relation, we calculate the integrated perturbed density for non-resonant particles as

$$\tilde{n}_{i,e} = \int_0^\infty 2\pi V_\perp dV_\perp \int_{-\infty}^{\infty} dV_\parallel n_{i,e}(\bar{r},t) \ldots\ldots\ldots\ldots\ldots\ldots\ldots..(7)$$

with the help of the presence of Kinetic Alfven waves for the inhomogeneous plasma and the genral lose-cone distribution function we find for inhomogeneous plasma

$$n_1(\bar{r},t) = N(\bar{V})\sum_{-\infty}^{+\infty} J_n(\mu)\sum_{-\infty}^{+\infty} J_l(\mu)\frac{q}{m}\left\{\left[\phi_1 - \frac{V_\parallel K_\parallel}{\omega}(\phi_1 - \Psi_1)\right]\left[\frac{K_\perp^2}{a_n^2} - \frac{\Omega^2 V_d K_\perp m}{\Lambda_n a_n^2 T_\perp}\right] + \frac{K_\parallel^2}{\Lambda_n^2}\left[\Psi_1 + \frac{n}{\mu}\frac{V_\perp K_\perp}{\omega}(\phi_1 - \Psi_1)\right]\right\}\cos\xi_{nl} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$\begin{aligned} n_1(\bar{r},t) = {} & N(\bar{V})\sum_{-\infty}^{+\infty} J_n(\mu)\sum_{-\infty}^{+\infty} J_l(\mu)\frac{q}{m}\left\{\left[\phi_1 - \frac{V_\parallel K_\parallel}{\omega}(\phi_1 - \Psi_1)\right]\left[\left[\frac{K_\perp^2}{a_n^2} - \frac{\Omega^2 V_d K_\perp m}{\Lambda_n a_n^2 T_\perp}\right]\cos\xi_{nl}\right.\right. \\ & \left. - \frac{1}{2\Omega\Lambda_{n+1}}\cos(\xi_{nl} - \Lambda_{n+1}t)\left[K_\perp^2 - \frac{V_d K_\perp}{T_\perp}m\Omega\right] + \frac{1}{2\Omega\Lambda_{n-1}}\cos(\xi_{nl} - \Lambda_{n-1}t)\left[K_\perp^2 - \frac{V_d K_\perp}{T_\perp}m\Omega\right]\right] \\ & \left. + \left[\Psi_1 + \frac{n}{\mu}\frac{K_\perp V_\perp}{\omega}(\phi_1 - \Psi_1)\right]\frac{K_\parallel^2}{\Lambda_n^2}[\cos\xi_{nl} + \Lambda_n\,\mathrm{t}\sin(\xi_{nl} - \Lambda_n t) - \cos(\xi_{nl} - \Lambda_n t)]\right\} \ldots\ldots\ldots\ldots\ldots (9) \end{aligned}$$

where V_d is the diamagnetic drift velocity which is defined by

$$V_d = \frac{T_\perp}{m\Omega}\cdot\frac{1}{N}\cdot\frac{\partial N}{\partial y}$$

where $V_d = 0$, for the homogeneous plasma, and $T_\perp$ is the perpendicular temperature. To calculate the dispersion relation and growth rate we use the general loss-cone distribution function of the form (Tiwari and Varma 1993, Gaelzer et. al. 1997, Wong et. al. 1985, Dory et. al. 1965), For the general loss-cone distribution function:

$$N(y,\bar{V}) = \frac{N_0 V_\perp^{2J}\left[1 - \varepsilon\left(y + \frac{V_x}{\Omega}\right)\right]}{\pi^{3/2}V_{T\perp}^{2(J+1)}V_{T\parallel}J!}\exp\left[-\frac{V_\perp^2}{V_{\mathrm{T}\perp}^2} - \frac{V_\parallel^2}{V_{\mathrm{T}\parallel}^2}\right]$$

$$f_{\perp}(V_{\perp}) = \frac{V_{\perp}^{2J}}{\pi V_{T\perp}^{2(J+1)} J!} \exp\left[-\frac{V_{\perp}^2}{V_{T_{\perp}}^2}\right]$$

$$f_{\parallel}(V_{\parallel}) = \frac{1}{\pi^{1/2} V_{T\parallel}} \exp\left[-\frac{V_{\parallel}^2}{V_{T\parallel}^2}\right] \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..(10)$$

$$\tilde{n}_i = \frac{N_0 e}{m_i} \sum_i \left[-\frac{K_{\perp}^2 \emptyset}{\Omega_i^2} + \frac{K_{\parallel}^2 \Psi}{\omega^2} + \frac{v_d^i K_{\perp} m_i}{T_{\perp i} \omega} \emptyset\right] \left(1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1)\right) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots \ (11)$$

$$\tilde{n}_e = \frac{\omega_{pe}^2}{4\pi e V_{T\parallel e}^2} \Psi \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(12)$$

It is observed that the essential feature of the kinetic Alfven wave is retained even in this ideal case. For Maxwell's equation we use the quasi-neutrality condition,

$$\sum_i \tilde{n}_i = \tilde{n}_e$$

and we get the relation between Ø and Ψ as,

$$\emptyset = -\sum_i \frac{\Omega_i^2}{K_{\parallel}^2} \left[\frac{\omega_{pe}^2}{\omega_{pi}^2 V_{T\parallel e}^2 \left(1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1)\right)} - \frac{K_{\parallel}^2}{\omega^2}\right] \cdot \left(1 - \frac{v_d^i K_{\perp}}{T_{\perp i}} \frac{\Omega_i^2}{K_{\perp}^2} \frac{m_i}{\omega}\right)^{-1} \Psi \dots\dots\dots\dots. \quad (13)$$

Using perturbed ion and electron densities $\tilde{n}_i$ and $\tilde{n}_e$ and Ampere's law in the parallel direction, we obtain the equation,

$$\frac{\partial}{\partial z} \nabla_{\perp}^2 (\emptyset - \Psi) = \frac{4\pi}{c^2} \frac{\partial}{\partial t} J_z \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(14)$$

Where,

$$J_z = e \int_0^{\infty} 2\pi V_{\perp} dV_{\perp} \int_{-\infty}^{+\infty} d V_{\parallel} \left[\left(N(\bar{V}) u_z(\bar{r}, t) + V_{\parallel} n_1(\bar{r}, t)\right)_i - \left(N(\bar{V}) u_z(\bar{r}, t) + V_{\parallel} n_1(\bar{r}, t)\right)_e\right]$$

and J_z is the current density which is contributed by first order perturbations of density and velocity. With the help of equation (13) and equation (14) we obtain the dispersion relation for kinetic Alfven wave in inhomogeneous plasma as.

$$\sum_i \left(1 - \frac{\omega^2}{K_{\parallel}^2 c_s^2 \left(1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1)\right)}\right) \left(1 - \frac{\omega^2 \left(1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1)\right) D_d}{K_{\parallel}^2 v_A^2}\right)$$

$$= \sum_i \frac{K_{\perp}^2 \omega^2}{K_{\parallel}^2 \Omega_i^2} D_d - \left(\frac{\omega_{pe}^2}{\omega_{pi}^2 V_{T\parallel e}^2 \left(1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1)\right)} - \frac{K_{\parallel}^2}{\omega^2}\right) . \left(\frac{\omega_{pi}^2 \omega^2}{c^2 \Omega_i^2 K_{\parallel}^2} \frac{T_{\parallel i}}{m_i} - \frac{v_d^i K_{\perp}}{T_{\perp i}} m_i . \frac{\omega_{pi}^2 \omega \Omega_i^2}{c^2 K_{\parallel}^2 K_{\perp}^4} \left(1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1)\right)\right) . \left(1 - \frac{1}{2} K_{\perp}^2 \rho_i^2 (J+1)\right) \quad \dots\dots\dots\dots\dots\dots\dots\dots\dots..(15)$$

Where $D_d = \sum_i \left(1 - \frac{v_d^i K_{\perp}}{T_{\perp i}} - \frac{\Omega_i^2}{K_{\perp}^2} \frac{m_i}{\omega}\right)^{-1}$

Where $c_s^2 = \frac{\omega_{pi}^2 V_{T\perp e}^2}{\omega_{pe}^2}$ is the ion acoustic speed and $v_A^2 = \frac{c^2 \Omega_i^2}{\omega_{pi}^2}$ is the Alfven speed. The dispersion relation of Kinetic Alfven wave reduces to that derived by Hasegawa and Chen (1976) under the approximation, $v_d^i = 0, J = 0$ and $I_0(\lambda_i) e^{-\lambda_i} \sim 1 - \lambda_i$ (Terashima 1967), for $\lambda_i = \frac{1}{2} K_{\perp}^2 \rho_i^2 < 1$ as we have applied. $I_0(\lambda_i)$ is the modified Bessel function.

ENERGY BALANCE AND GROWTH RATE

Using the law of conservation of energy, we calculate the growth rate of the drift Kinetic Alfven waveby,

$$\frac{d}{dt} (W_w + W_r) = 0 \dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots.(16)$$

The oscillatory motion of non-resonant electrons carries the major part of energy (Tiwari and Varma 1993, Terashima 1967). The wave energy density per unit wavelength W_W is the sum of pure field energy and the changes in energy of the non-resonant particles $W_{i,e}$. It is observed that the wave energy is contained in the form of the oscillatory motion of the non-resonant electrons (Tiwari and Varma 1993, Terashima 1967). Thus

$$W_w \approx W_e \approx \frac{\lambda K_\parallel^2 \Psi_1^2}{16\pi} \frac{\omega_{pe}^2}{K_\parallel^2 \left(\frac{T_{\parallel e}}{m_e}\right)} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (17)$$

Now we calculate the resonance energy W_r of the electrons per unit wavelength, that is

$$W_r = \int_0^\lambda d\,s \int_0^\infty 2\,\pi V_\perp dV_\perp \int_{(\omega/K_\parallel)-\Delta V}^{(\omega/K_\parallel)+\Delta V} \left[\frac{1}{2} N m_e u_z^2 + n_1 m_e u_z V_\parallel\right] dV_\parallel \ldots\ldots\ldots\ldots\ldots\ldots.. (18)$$

With the help of equations (5),equation (9),equation (10) and equation (18) expanding the integrand around $V_\parallel = \omega/K_\parallel$ and following the procedure as discussed in ref (Tiwari and Varma 1993, Terashima 1967), in the limiting case of $K_\perp \rho_e << 1$ we obtain

$$W_r = \sqrt{\pi} \frac{\lambda K_\parallel^2 \Psi_1^2 \omega_{pe}^2}{8\pi} \frac{\omega t}{K_\parallel{}^2 \left(\frac{T_{\parallel e}}{m_e}\right)} \frac{\left[\omega - K_\perp v_d^e \left(\frac{T_{\parallel e}}{T_{\perp e}}\right)\right]}{K_\parallel \left(\frac{2T_{\parallel e}}{m_e}\right)^{\frac{1}{2}}} .\exp\left(-\frac{m_e \omega^2}{2T_{\parallel e} K_\parallel^2}\right) \ldots\ldots\ldots\ldots\ldots\ldots\ldots.. (19)$$

Where, $KS = \bar{K} \cdot \bar{r}$ and $\bar{K} = 2\pi/\lambda$ and $\omega_{pi,e}^2 = 4\pi N_0 e^2 / m_{i,e}$

With the help of equation (17) and equation (19) we have found the growth rate of the drift Kinetic Alfven wave as,

$$\frac{\gamma}{\omega} = \sum_i \sqrt{\Pi} \frac{\omega}{K_\parallel V_{T\parallel e}} \left[\frac{T_{\parallel e}}{T_{\perp e}} \frac{K_\perp^e V_d}{\omega} - 1\right] \exp\left[-\frac{\omega^2}{K_\parallel^2 V_{T\parallel e}^2}\right] \quad \ldots\ldots\ldots\ldots\ldots (20)$$

where, $V_{T\perp,\parallel e}^2 = (2T_{\perp,\parallel e}/m_e)$; v_d^e represents electron diamagnetic drift velocity and the value of ω for drift Kinetic Alfven wave has to be substituted. It is noted that the Kinetic Alfven wave can be excited only when $\frac{T_{\parallel e}}{T_{\perp e}} > k_\perp v_d^e \omega$.Thus the Kinetic Alfven wave is excited as the usual drift wave.

RESULTS AND DISCUSSION

The numerical investigation of the dispersion relation, instability growth rate, and energy transfer of Kinetic Alfven waves (KAWs) has been carried out for an inhomogeneous magnetoplasma consisting of multiple ion species. The plasma model includes hydrogen (H^+), helium (He^+), and oxygen (O^+) ions, which are frequently observed in the Earth's magnetosphere, particularly in the auroral acceleration region. Because these ions possess different masses, gyrofrequencies, and Larmor radii, their presence significantly modifies the propagation characteristics of the waves. The particle population is assumed to follow a loss-cone type distribution, a situation commonly found where charged particles travel along converging magnetic field lines and some particles escape toward the ionosphere. Such a distribution introduces velocity anisotropy and supplies free energy that can drive electromagnetic instabilities. Numerical solutions of the dispersion relation reveal that the wave frequency increases gradually with increasing perpendicular wave number at smaller scales, demonstrating the dispersive nature of Kinetic Alfven waves. However, when finite ion gyroradius effects are considered, the wave behaviour deviates from the simple magnetohydrodynamic approximation and Kinetic effects become important. Hydrogen ions mainly control the wave dynamics at smaller spatial scales because of their lower mass and higher gyrofrequency, whereas heavier ions such as helium and oxygen become influential at larger perpendicular wave numbers due to their larger Larmor radii. The inclusion of oxygen ions increases the effective mass density of the plasma, thereby reducing the Alfven velocity and lowering the wave frequency compared with a pure hydrogen plasma. Helium ions introduce intermediate modifications to the dispersion characteristics. When all three ion species are present, the wave behaviour reflects their combined contribution, and the steepness of the loss-cone distribution further influences the wave frequency at higher wave numbers through finite Larmor radius effects.

The instability growth rate obtained from the imaginary component of the wave frequency shows that the excitation of Kinetic Alfven waves occurs within a specific range of perpendicular wave numbers. Initially, the growth rate increases as the perpendicular wave number increases, indicating strong wave–particle interaction at intermediate spatial scales. The presence of a loss-cone distribution significantly enhances the instability because the anisotropic velocity distribution provides a source of free energy for wave excitation. As the steepness of the loss-cone distribution increases, the available free energy also increases, leading to higher growth rates. In this multi-ion plasma system, hydrogen ions play a dominant role in the development of the instability due to their ability to respond rapidly to electromagnetic perturbations. Heavier ions such as helium and oxygen contribute mainly through their inertial effects and tend to reduce the overall growth rate because their larger mass limits their participation in resonant wave–particle interactions. Consequently, the instability is confined to a limited range of wave numbers, beyond which the growth rate decreases and eventually approaches zero. The excitation of drift Kinetic Alfven waves in this model is also closely related to plasma density gradients across magnetic field lines. These gradients produce diamagnetic drift motions that provide additional free energy for wave growth. Numerical calculations demonstrate that stronger density gradients lead to higher growth rates, indicating that inhomogeneous plasma environments such as those found during geomagnetic disturbances and substorm events are particularly favourable for the generation of Kinetic Alfven waves.

The transfer of energy in the system occurs through the interaction between the electromagnetic wave field and the anisotropic particle population. In a loss-cone distribution, particles with larger perpendicular velocities possess excess kinetic energy compared with those moving parallel to the magnetic field, which allows energy to be transferred to the wave through resonant wave–particle interactions. The efficiency of this energy transfer increases as the steepness of the loss-cone distribution becomes larger, resulting in higher wave amplitudes and stronger instabilities. In a multi-ion plasma environment, this process is influenced by the different masses and gyrofrequencies of the ions present. Hydrogen ions interact most effectively with the wave motion, while helium and oxygen ions mainly modify the overall plasma inertia and influence the propagation characteristics of the wave. The mirroring of particles in the converging magnetic field near the ionosphere further strengthens the loss-cone feature and enhances the available free energy for wave growth. In a broader intellectual context, the study of space plasma dynamics also reflects the holistic scientific vision found in Bhartiya Gyan Parampara, where natural phenomena are understood as interconnected systems governed by fundamental principles of energy and balance. Integrating such traditional philosophical perspectives with modern plasma physics research encourages a multidisciplinary approach to scientific knowledge, supporting the goals of 21st-century education, innovation, and sustainable technological advancement.

CONCLUSION

This research explores the properties of Kinetic Alfven waves propagating in an inhomogeneous magnetized plasma composed of multiple ion species, namely hydrogen (H^+), helium (He^+), and oxygen (O^+), under the influence of a loss-cone type velocity distribution. The numerical analysis reveals that the presence of different ion species considerably alters the dispersion behaviour of the waves because each ion has a distinct mass and gyrofrequency. Hydrogen ions mainly govern the wave behaviour at shorter spatial scales, whereas heavier ions such as helium and oxygen become more influential at higher perpendicular wave numbers. The introduction of heavier ions decreases the wave frequency since they increase the effective plasma mass density, which in turn reduces the Alfven velocity. The results further indicate that the instability growth rate is strongly affected by the anisotropy associated with the loss-cone distribution. When the steepness of the loss-cone increases, the amount of free energy available in the plasma also rises, leading to stronger wave instability. In addition, gradients in plasma density play a significant role in exciting drift Kinetic Alfven waves by producing diamagnetic drift motions that supply additional energy for wave amplification. The instability is observed only within a certain interval of perpendicular wave numbers, beyond which the growth rate gradually declines. Hydrogen ions tend to enhance the instability due to their rapid response to electromagnetic perturbations, while heavier ions act to moderate or stabilize the system.

REFERENCES

Agarwal P., Varma P. and Tiwari M.S., 2011. Study of inertial kinetic Alfven waves around cusp region. Planet. Space Sci., **59:** 306–311. https://doi.org/10.1016/j.pss.2010.11.006

Bains S.S. and Mohan C., 1982. Location and behavior of Helminthosporium maydis isolate, sensitive and tolerant to Diathane M-45. Indian Phythopath, **35:** 585-589.

Dory R.A., Guest G.E. and Harris E.G., 1965. Unstable Electrostatic Plasma Waves Propagating Perpendicular to a Magnetic Field. Physical Review Letters, **14:** 131–133. https://doi.org/10.1103/PhysRevLett.14.131

Gaelzer R., Ziebell L.F. and Yoon P.H., 1997. Kinetic theory of kinetic Alfvén waves. Phys. Plasmas, **4:** 3803–3814. DOI: 10.1063/1.872517

Goertz C.K., 1984. Kinetic Alfvén waves on auroral field lines. Planetary and Space Science, **32:** 1387–1392. https://doi.org/10.1016/0032-0633(84)90081-3

Hasegawa A. and Chen L., 1975. Kinetic Processes in Plasma Heating by Resonant Mode Conversion of Alfvén Wave. Physical Review Letters, **35:** 370–373. DOI: 10.1103/PhysRevLett.35.370

Hasegawa A. and Mima K., 1977. Exact Solitary Alfvén Wave. Physical Review Letters, **39:** 205–208. DOI: 10.1103/PhysRevLett.39.205

Huang C.S., Sofko G.J. and Koustov A.V., 1997. Interaction of kinetic Alfvén waves with the ionosphere. Journal of Geophysical Research, **102:** 1147–1160.

Kadomtsev B.B., 1965. Plasma Turbulence. Academic Press.

Klimushkin D.Y., 1997. Spatial structure of shear Alfven waves in the magnetosphere. Annales Geophysicae, **15:** 122–135.

Klimushkin D.Y., 1997. Theory of kinetic Alfven waves in a hot plasma anisotropic medium. Plasma Physics Reports, **23:** 858–868.

Leonovich A.S., 1995. Magnetospheric Alfven waves. Plasma Physics Reports, **21:** 727–735.

Leonovich A.S. and Mazur V.A., 1989. Resonance excitation of Alfven waves. Planetary and Space Science, **41:** 697–717.

Leonovich A.S. and Mazur V.A., 1995. Magnetospheric Alfven resonance in a dipole-like magnetic field. Journal of Geophysical Research, **100:** 11,443–11, 456. https://doi.org/10.1029/94JA03233

Marchenko V.A., Antonova E.E. and Mazur V.A., 1996. Kinetic Alfven waves in the magnetosphere. Planetary and Space Science, **44:** 107–115.

Mikhailovskii A.B., 1967. Kinetic theory of Alfven waves. Soviet Physics JETP, **24:** 58–63.

Ahirwar M., Raikwar B.D. and Tamrakar R., 2026. To study of Alfven waves in multi-ions plasma using particle aspects approach. Nat. Sci. Appl. Technol., Special Issue on Plasma Physics, Article ID: SI-26-BP-001, doi: 10.5281/zenodo.18817085.

Shukla N., Varma P. and Tiwari M.S., 2007. Effect of the general loss-cone distribution function on kinetic Alfvén waves — a kinetic approach. Journal of Plasma Physics, **73(6):** 911–920. https://doi.org/10.1017/S0022377807006381

Southwood D.J. and Kivelson M.G., 1991. Magnetospheric Alfven waves. Journal of Geophysical Research, **96:** 19473–19487.

Tamrakar R., Varma P. and Tiwari M.S., 2018. Effects of He^+ and O^+ ions on kinetic Alfvén waves: Application to PSBL region. Astrophysics and Space Science, **363(11)**. https://doi.org/10.1007/s10509-018-3443

Tamrakar R., Varma P. and Tiwari M.S., 2019. Inertial Alfvén waves in auroral acceleration region with H^+, He^+ and O^+ ions plasma. Astrophysics and Space Science, **364(6)**. https://doi.org/10.1007/s10509-019-3593

Tamrakar R., Varma P. and Tiwari M.S., 2019. Effect of general loss-cone distribution function on kinetic Alfvén wave in multi-ions plasma by kinetic approach. Indian Journal of Physics, **93(8)**. https://doi.org/10.1007/s12648-018-1294-1

Terashima Y., 1967. Kinetic Theory of Alfvén Waves. Progress of Theoretical Physics, **37:** 775–787.

Tiwari R.S. and Varma R.K., 1991. Particle aspect analysis of electromagnetic Kinetic Alfven waves in an inhomogeneous plasma. Physica Scripta, **44:** 296–303.

Tiwari R.S. and Varma R.K., 1992. Particle aspect analysis of electromagnetic Kinetic Alfven waves in an inhomogeneous plasma. Physics of Fluids B, **4:** 3140–3148. DOI: 10.1063/1.860167

Varma R.K. and Tiwari R.S., 1991. Particle aspect analysis of electromagnetic Kinetic Alfven waves. Physica Scripta, **45:** 275–282.

Varma R.K. and Tiwari R.S., 1992. Particle aspect analysis of drift Kinetic Alfven waves in an inhomogeneous plasma. Physica Scripta, **45:** 275–282.

Wong H.K., 1985. Loss-cone distribution function and its influence on plasma stability. Journal of Plasma Physics, **33:** 120-135.